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Space-Time Dual Geometry Theory of Elementary Particles and Their Interaction Fields[1]

Jaime Keller

División de Estudios de Posgrado, Facultad de Química, Universidad Nacional Autónoma de México, 04510 México, D.F.

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The important role of the operator γ_5 in the physics of elementary particles and their interactions is considered basic in this paper where it is shown that it corresponds to the duality rotation in space-time and how the dual geometry can be used to consider new symmetries for the wave equations of particles and their interaction fields. The new set of symmetries is shown to exactly correspond to what is needed to find the known schemes of leptons and quarks and to be the origin of many of the properties which are found experimentally for these elementary particles.

1. INTRODUCTION

What is known at present about elementary particles, leptons, quarks, and Higgs fields has enabled high-energy physicists to construct a periodic table where almost three families are now contained. Group theoretical studies are telling us which are the basic symmetries of the particles and their interactions. The present paper studies this problem from a different point of view: which can be the origin, within the frame of the physical space-time, of this scheme?

A suitable theory should be able not only to accommodate the known quarks, leptons, and Higgs fields, and to provide a basis for the study of their interactions, most probably in the form of gauge fields, but also of explaining other constraints like the ones used in the Weinberg (1967) and Salam (1968) theory of weak interactions allowing left-handed fields only,

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although the electron may also have a non-weakly-interacting, right-handed field. The basic operator to project out left- and right-handed fields $(1 \mp i\gamma_5)$ is also used in quantum mechanics of spin-1/2 particles to transform the operator for a vector γ_μ into the operator for an axial vector $\gamma_5\gamma_\mu$. Vectors and axial vectors are duals in the geometry generated by the vectors. As the γ_μ vectors generate space-time geometry, the duality operation actually corresponds to duality in space-time. This tells us that one of the basic symmetries of matter fields must be dual geometry symmetry.

In this paper, after a brief review of space-time geometry, we discuss Lorentz and duality transformations, their implications for quantum mechanics, and the wave equations for dual geometry constrained particles and their interaction fields. At the end we discuss the connection with what is experimentally known about leptons and quarks.

2. SPACE-TIME GEOMETRY

The physical space-time corresponds to the vector space where the physical events occur. Physical events are primitive notions, together with the idea that they form series, or physical timelike trajectories, and that there exists some order in the series such that for each event series, or world trajectory, there is a "past" and a "future" to a present event. Then the physical space-time defines a geometry as the set of all possible physical events such that world timelike trajectories with geometrical order are contained. The basic trajectories are taken to be light signals used to define the causal relationships between the events. The quantum nature of the physical events is also accommodated in the scheme, as we will do in this paper, considering matter particles as sets of nondecaying events associated to a hypervolume h. This will provide a basis for a consistent theory of matter where the known elementary particles correspond to the different ways a field can be constructed, which is compatible with the postulates above. A series of trajectories are available, for a given matter particle, where the events are then given a nondecaying phase.

The physical space-time relates pairs of these events in such a causal way that for an event a_0 for particle **a** of trajectory **A**, the trajectory **B** of the series of events of particle **b** is divided in three segments: the absolute past $\mathbf{B}_1$, the non-causally-connected $\mathbf{B}_2$, and the absolute future $\mathbf{B}_3$ such that, if $\rightarrow$ means "follows in the time-ordered sense," $\{b_1 \in \mathbf{B}_1\} \rightarrow \{b_2 \in \mathbf{B}_2\} \rightarrow \{b_3 \in \mathbf{B}_3\}$, an event b_1 can influence but not being influenced by a_0, b_2 cannot influence or be influenced by a_0, and b_3 can be causally influenced by a_0, but cannot influence a_0. Moreover $\mathbf{B} = \mathbf{B}_1 \cup \mathbf{B}_2 \cup \mathbf{B}_3$.

We also require that if b'_3 and b''_3 are in the future of **a**, that is, if there is a free particle or light trajectory from **a** to b'_3 and b''_3 and there is a point d'_3 in the future of some $b \rightarrow b'_3 \rightarrow b''_3$ such that d'_3 is in the (**a** to b'_3) trajectory, then the (b to d) trajectory crosses the (**a** to b''_3) trajectory at some d''_3 such that $b \rightarrow d'_3 \rightarrow d''_3$.

The particles and light trajectories are considered to be continuous and equivalent for change of orientation in the sense that if the double trajectory (**a** to b_3 to a_3) is physically equivalent to the (**a** to c_3 to a_3) then one can be obtained from the other by a Lorentz group rotation. The physical space-time is finally considered to be four dimensional.

It is important to recognize that there are no one-dimensional, two-dimensional, or three-dimensional physical spaces but only physical series of events where the equations are independent of three-, two-, or one-coordinate parameters.

Lower- or higher-dimensional physics seem to be nothing more than useful exercises. The definition of light trajectories as the fastest ones and the limit for physical phenomena trajectories puts them in a special position as geodesics in space-time for which there is no perpendicular space. The speed of light is a constant, a global constant in Minkowski space-time and a local constant in general relativity where the Lorentz invariance is also generalized to a local symmetry; the geometry generated with these properties constitutes the physical space-time.

Light trajectories are the fastest trajectories in the sense that they mark the onset of absolute future for any physical event.

We now consider the tangent space $D(x)$ at point x of space-time with orthonormal vectors

$$\gamma_\mu = \Box x_\mu; \qquad \gamma_0^2 = 1, \qquad \gamma_1^2 = \gamma_2^2 = \gamma_3^2 = -1 \tag{1}$$

here $\Box$ corresponds to the gradient operator with components

$$\Box = \gamma_\mu \Box^\mu = \gamma^\mu \Box_\mu \tag{2}$$

where we have introduced a metric given by the symmetric product ($\cdot$ product)

$$g_{\mu\nu} = \gamma_\mu \cdot \gamma_\nu \equiv \tfrac{1}{2}(\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu) \tag{3}$$

we can also define contravariant multivectors from their antisymmetric product (this product also called Λ product) (Proca, 1930a, 1930b; Sauter, 1930a, 1930b; Juvet, 1930, 1932; Eddington, 1936; Mercier, 1934, 1935; Sommerfeld, 1939; Ravševskiĭ, 1957; Riesz, 1946, 1953, 1958; Teitler, 1965a, 1965b, 1965c, 1966a, 1966b; Hestenes, 1966, 1975; Salingaros and Dresden, 1979; Greider, 1980; Casanova, 1970, 1976; Boudet, 1971, 1974; Quilichini, 1971), as

$$\gamma_{\mu\nu\ldots\lambda} \equiv \gamma_\mu \Lambda \gamma_\nu \Lambda \ldots \Lambda \gamma_\lambda \tag{4}$$

defined by recursion that is, if b is an n-vector and a is a 1-vector,

$$a\Lambda b \equiv \tfrac{1}{2}(ab + (-1)^n ba) = a \Lambda b_1 \Lambda b_2 \Lambda \ldots \Lambda b_n \tag{5}$$

or, in general

$$A \Lambda B = \tfrac{1}{2}(AB - \bar{B}\bar{A}) \tag{6}$$

for any pair of multivectors A and B, where $\bar{A}$ and $\bar{B}$ are the reverse multivectors, it is a multivector where the sign of all vectors has been reversed. The full power of multivector algebra is obtained if, following Mercier (1935) and Hestenes (1966), we define their total or geometrical product

$$ab = a \cdot b + a \Lambda b \tag{7}$$

where the generalized inner product is given by

$$A \cdot B = \tfrac{1}{2}(AB + \bar{A}\bar{B}) \tag{8}$$

The gradient operator has the two following main properties (Hestenes, 1966): (a) $\Box_i$ maps scalars ϕ into scalars,

$$\Box_i \phi = \partial_i \phi \tag{9}$$

(b) $\Box_i$ maps vectors into vectors,

$$\Box_i \gamma_j = -\Gamma_{ij}^k \gamma_k \tag{10}$$

This defining the connection coefficients Γ_{ij}^k for the frame $\{\gamma_k\}$. It obeys the Leibnitz rule and is distributive with respect to addition.

An observer S_1 at x has at his disposal the 16 different basic multivectors of the space-time algebra, also called Dirac algebra. This algebra and its symmetries is shown in Figure 1. It should be remembered that all the elements of the space-time geometric algebra also correspond to operators, Figure 1 shows this correspondence where **1** is the identity operation, $\mathbb{P}$ corresponds to the parity operation, $\mathbb{T}$ to the time reversal operator, $\mathscr{L}$ to

SPACETIME GEOMETRY

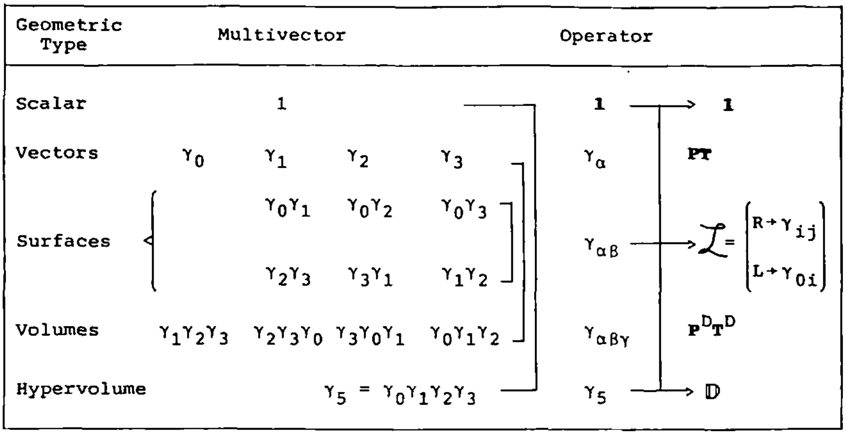

Fig. 1. Space-time multivectors, their symmetries and their properties as operators.

the Lorentz group of rotations R and Lorentz boosts L, $\mathbb{D}$ corresponds to the duality operator. The super index D to the dual of the operator. The duals of the scalars are the hypervolumes, the duals of the vectors are the volumes or axial vectors, and the duals of the spacelike surfaces are the timelike surfaces; this is also shown in Figure 1. Geometric duality is a reciprocal property and in practice it has the geometrical meaning that the product of a quantity and its dual corresponds to hypervolume or space-time volume.

3. LORENTZ TRANSFORMATION AND DUALITY ROTATIONS

A Lorentz transformation of one member of the Dirac algebra $\mathscr{D}(x)$ takes the form

$$\mathscr{D}(x) \to \mathscr{D}'(x) = R(x)\mathscr{D}(x)R^{-1}(x) \tag{11}$$

with R being the sum of a scalar, a bivector, and a pseudoscalar such that

$$RR^{-1} = R^{-1}R = 1 \qquad \text{and} \qquad R^{-1} = \pm \tilde{R} \tag{12}$$

($\tilde{R}$ is a multivector where the order of all products has been reversed) which is written in terms of generators

$$R = \pm e^{-1/2B}; \qquad B = b^{\mu\nu}\gamma_\mu\gamma_\nu, \qquad \mu \neq \nu \tag{13}$$

with B corresponding to a bivector as shown in Figure 1. Then the vectors associated with the second observer S_2

$$\alpha_\mu = R\gamma_\mu R^{-1} \tag{14}$$

will again be orthogonal but not necessarily constant.

A new type of operation, the duality rotation $\mathbf{D}_\mu$ is generated by the operator $\gamma_5 t^\mu \gamma_\mu \gamma_\mu \cdot$, where the dyad $\gamma_\mu \gamma_\mu \cdot$ has been introduced to allow independent rotations of the four basis vectors; this operation is continuously connected to unity through the duality rotation operation

$$\mathbf{D}_\mu = \exp(i\gamma_5 t^\mu \gamma_\mu \gamma_\mu \cdot), \qquad (i\gamma_5)^2 = 1 \tag{15}$$

4. QUANTUM MECHANICS

To study a physical system consider the energy momentum vector relation $\mathbf{p} = \mathbf{p}'$ for a "particle" moving with respect to observer S_1,

$$p^0\gamma_0 + p^1\gamma_1 + p^2\gamma_2 + p^3\gamma_3 = p'^0\gamma_0' + p'^1\gamma_1' + p'^2\gamma_2' + p'^3\gamma_3' \tag{16}$$

the primed components are those computed by S_1 when the particle's motion is referred to an S_1' inertial system. A similar result could be obtained if a three-vector relation $\mathsf{p} = \mathsf{p}'$ is used instead, with the same components (Keller, 1982b), with the added feature that the three vectors $\gamma_{\mu\nu\rho}$ are now "oriented" with respect to observer S_1 and two types of "particles" will now be possible those with positive orientation which could be interpreted as positive mass particles or standard particles, and those of negative orientation, which could be interpreted as negative mass particles or antiparticles. The action $\mathsf{p}\cdot\mathsf{x}$ being an hypervolume.

A Lorentz transformation R relates S_1 and S_1' in such a way that

$$\gamma_\alpha' = R\gamma_\alpha\tilde{R} \qquad \text{and} \qquad \gamma_\alpha = \tilde{R}\gamma_\alpha' R \tag{17}$$

and, substituting the first of these relations into (16), we obtain

$$p^\alpha\gamma_\alpha = p'^\alpha R\gamma_\alpha\tilde{R} \tag{18}$$

With this transformation the explicit dependence on the vectors of S_1' has disappeared, in fact equations (18) and (19) below are written in terms of the vectors of S_1.

Here we will do two substitutions which will transform this equation (18) into a multivector form of the Dirac equation (Keller, 1982a), first multiplying by R (constructed with vectors of S_1) on the right, to obtain

$$p^{\alpha}\gamma_{\alpha}R = p'^{\alpha}R\gamma_{\alpha} \tag{19}$$

We may then make a further Lorentz transformation with the even multivector Q which is a generalized gauge transformation

$$Q = \exp(-I\{\mathbf{p}\cdot\mathbf{x} + \phi(\mathbf{x}) + i\gamma_5[\mathbf{p}'\cdot\mathbf{x}' + \phi'(\mathbf{x}')]\}/\hbar), \qquad I^2 = -1 \tag{20}$$

the first term in the exponent with a particular choice (standard representation) $I = \gamma_{12} = \gamma_5\gamma_0\gamma_3$ which corresponds to a rotation in an arbitrary plane. The phase factors $\phi(\mathbf{x})$ generate a (set of) connection(s). These factors are, in general, the sum of a scalar, a bivector, and a pseudoscalar and, as we will see in the next two parts, they will correspond to gauge interactions fields generating the electromagnetic, weak, gluon, and gravitational potentials. Otherwise $\phi(\mathbf{x})$ and $\phi'(\mathbf{x})$ should be taken as constants. The $\phi(\mathbf{x})$ have to be even too.

This factor Q, a further Lorentz transformation, can be used, with constant ϕ and ϕ', to obtain the values of the components p^{α} using $g_{\alpha\beta}\partial_{\beta}QI = p^{\alpha}Q$. Define now the composite Lorentz transformation

$$\psi = RQ, \qquad \tilde{\psi} = \tilde{Q}\tilde{R} \tag{21}$$

to obtain (Keller, 1982a, b) the eigenvalue equation

$$\gamma_{\alpha}g_{\alpha\beta}\partial_{\beta}\Psi I = ig_{\alpha\beta}\partial'_{\alpha}\Psi\gamma_5 I\gamma_{\beta} \tag{22}$$

which we called the full Dirac equation, in multivector form.

Equation (22) shows that we could mix in the same equation γ_{α} and $\gamma_5\gamma_{\alpha}$ that is, a vector and its geometrical dual the axial vector $\gamma_5\gamma_{\alpha} = \gamma_{\alpha}^{D}$. A simpler, equally useful although less symmetrical equation, is obtained if we use the more common expression $p^{\alpha}\gamma_{\alpha} = m_0\gamma'_0$ as a starting point in (16) to obtain, instead of (22), the equation

$$\gamma_{\mu}g_{\alpha\mu}\partial_{\alpha}\psi I = m_0\psi\gamma_0 \tag{23}$$

which is the Dirac equation in multivector form. It was derived by Keller (1982a, b) without the use of the standard postulates of quantum mechanics. The analysis reproduced here constitutes a new way of obtaining basic operators of quantum theory. Otherwise the factor Q, equation (20), is the expression of the de Broglie's phase wave in the multivector form of quantum mechanics. The first time that an equation like (23) was used, corresponds to the work of Proca (1930a, b) but it was really obtained from the interpretation of the Dirac algebra as a geometric algebra by Hestenes (1966, 1975, and references therein). The work of Teitler (1965a, b, c; 1966a, b) and of Casanova (1970, 1976) should also be mentioned. Our derivation of (22) and (23) further justifies these equations. It shows a direct path from the de Broglie's hypothesis to the basic equations of quantum mechanics. The matrix representation of the γ_α vectors generates the matrix form of the Dirac equation. But in order to obtain the Dirac equation in the standard form, without a matrix representation, all that is needed is to define the wave functions ψ to be eigenfunctions of $\gamma_0 I$, with eigenvalue i [equation (23) must be multiplied by I on the right],

$$\psi\gamma_0 I = i\psi \tag{24}$$

reducing (23) to the standard Dirac equation in the Proca form. Another approach is to define a Dirac spinor u to be an eigenspinor of the same operator, $\gamma_0 Iu = iu$, the standard Dirac wave function being

$$\Psi = \psi u \tag{25}$$

with $\bar{u} = (1, 0, 0, 0)$ when the vectors γ_α are represented in the standard form.

The presence of the gauge phase factors $\phi(\mathbf{x})$ will make the transformations (17) nonconstant with the result that the local timelike vector of the particle's reference system

$$\rho_\mu(\mathbf{x}) \equiv (\gamma_0'(\mathbf{x}))_\mu = (\tilde{\psi}(\mathbf{x})\gamma_0\psi(\mathbf{x}))_\mu = \tilde{\psi}(\mathbf{x})\gamma_0\psi(\mathbf{x})\gamma_\mu \tag{26}$$

changes from one point of space-time to another. This allows a new type of question: what is the relative probability of presence for particle k in space-time volumes v_1 and v_2? This leads to a probabilistic analysis of the results obtained with the wave equations.

5. DUAL GEOMETRY THEORY OF ELEMENTARY PARTICLES

As we mentioned in the introduction, the operator to project left- and right-handed fields is the same operator which transforms a vector into an axial vector. The Weinberg–Salam theory showed the basic role of this operation in elementary particle physics; it is to be expected that the unification of fundamental interactions will not change this consideration, so then the unified scheme (Georgi and Glashow, 1974, Fritzsch and Minkowski, 1974, Georgi, 1975, Chanowitz, Ellis and Gaillard, 1977) joining strong and electro-weak interactions should clarify this particular property of the γ_5 operator or duality in the geometrical sense.

The duality operation will be introduced as a geometric duality transformation

$$\begin{aligned}\gamma_\mu &\to \gamma_\mu^K = \cos\theta_\mu \gamma_\mu + i\sin\theta_\mu \gamma_\mu^D \\ \gamma_\mu &\to i\gamma_\mu^{DK} = -\sin\theta_\mu \gamma_\mu + i\cos\theta_\mu \gamma_\mu^D\end{aligned} \tag{27}$$

or a multivector notation $\theta_\mu = t_\mu \theta$

$$\gamma_\mu \to \gamma_\mu^K = \exp\left(i\gamma_\mu^K t_\mu \theta \gamma_\mu \cdot /2\right)\gamma_\mu \cdot \exp\left(-i\gamma_\mu^K t_\mu \theta \gamma_\mu /2\right) = \exp\left(i\gamma_\mu^K t_\mu \theta \gamma_\mu \cdot\right)\gamma_\mu \tag{28}$$

as a continuous operation connected to the identity.

This duality transformation could be used in the exponent of the factor Q in equation (20) and correspondingly in equations (22) or (23). In the particular case of massless fields and $\theta = \pi/2$ as a reference duality rotation, we would obtain an equation

$$i\gamma_\beta g_{\beta\alpha}\partial_\alpha \Psi \gamma_5 \sin(t_\alpha \pi/2) + \gamma_\lambda g_{\lambda\alpha}\partial_\alpha \Psi \cos(t_\alpha \pi/2) = 0 \tag{29}$$

which is a combination of the standard Dirac equation for a massless field

$$i\gamma_\alpha g_{\alpha\beta}\partial_\beta \Psi = 0 \tag{30}$$

and its dual

$$\gamma_\alpha g_{\alpha\beta}\partial_\beta \Psi \gamma_5 = 0 \tag{31}$$

But it is not a simple combination because the duality rotation angle is allowed to have different values for different components α as shown by the subindex α in the coefficients t_α.

We could also write equation (29), using the definition of the K geometric duality transformation (27), in the simpler form

$$i\gamma_\alpha^K g_{\alpha\beta}\partial_\beta \Psi = 0 \tag{32}$$

This is possible because the wave function ψ is an even multivector, equation (13), then it commutes with γ_5.

If the duality rotation coefficients $t_\alpha = t$ for all four components, we could write on the right hand side of (29), (30) and (32) the mass term. That is, only if all four duality rotations are equal, massive fields are allowed.

Equations (29) (or (32)) are no longer Lorentz invariant except if we select the left-handed field (or the right-handed if $\theta = -\pi/2$),

$$\Psi_L\gamma_5 = -i\Psi_L, \qquad [\text{using } t_\mu = (t, t+1) = \text{integer in (29)}] \tag{33}$$

reducing equation (32) to (30). We may then see that the fact that in nature left-handed fields only are allowed, in certain cases (not for the electron where $t_\alpha = t+1$), makes the geometric duality rotation a basic operation of elementary particle physics. It also says that if experimentally left- and right-handed particles of a given type are found, they must correspond to fields where all four duality rotations are equal. If not all four duality rotations are equal, the fields must be massless, to begin with, and will not conserve Lorentz invariance unless a particular choice, left-handed field, say, is made.

But even if only left-handed fields are allowed, there is a further constrain on the t_α in equation (29). They should all be either equal or differ by only one unit among themselves. Otherwise, the substitution of (33) into (29) will not give (30) back. Then the physically allowed fields will be those for which the $t_\alpha = t,\ t+1$; reducing the number of physically allowed fields to a well-defined collection of sets. Each set corresponding to a given value of t. The values of t may only be $t = 0, 1, 2, 3$. Other sets will correspond to anyone of these basic ones as they will correspond up to a full 2π duality rotation. A special case is presented by the field where all four $t_\alpha = 0$, which we will treat separately in the following analysis.

The set of all physically allowed fields was given the generic name of symmetry *con*strained *Dirac* fields, or *diracons* for short (Keller, 1981, 1982a, b).

With the definitions and restrictions already discussed in this section we can construct four sets or families of which the first three are shown in Table I, $t = 0, 1, 2$, together with the possible identification of these fields. Each set should really be called a family generated by a value of t. It is seen that the collection of diracon fields can generate a unified description of quarks and leptons. In the next section the identifications will be further discussed when we analyze the gauge fields for the interactions, finding the correspondence with electroweak and color forces.

It is clear that dual geometry symmetry explains all the elementary particle fields known, the existence of color degeneracy for quarks, the existence of families and, as we will discuss in the next section, the origin of

TABLE I. Allowed Combinations of Quantum Numbers $a_\alpha = t_\alpha - t$ Corresponding to Duality Rotation of the Wave Function ψ of a Symmetry-Constrained Dirac Particle (Diracon)

a_1	a_2	a_3	a_0	Possible identification of the corresponding matter field $t = 0$ family	$t = 1$ family	$t = 2$ family
0	0	0	± 1	Electron neutrino	Muon neutrino	Tau neutrino
± 1	0	0	± 1	Parton ($t = 0$)	Parton ($t = 1$)	Parton ($t = 2$)
0	± 1	0	± 1	Three "colors"	Three "colors"	Three "colors"
0	0	± 1	± 1	($\bar{d}$ quark)	($\bar{s}$ quark)	($\bar{b}$ quark)
± 1	± 1	0	± 1	Parton ($t = 0$)	Parton ($t = 1$)	Parton ($t = 2$)
0	± 1	± 1	± 1	Three "colors"	Three "colors"	Three "colors"
± 1	0	± 1	± 1	(u quark)	(c quark)	(t quark)
± 1	± 1	± 1	± 1	Electron	Muon	Tau

the strength and confinement in color interactions. But dual geometry symmetry is not new in elementary particle physics or in the study of the interactions as discussed in the next paragraphs.

We need to make a short disgression about the electromagnetic field. It has been suggested that a natural extension of the Maxwell equations would be that where magnetic monopoles are accepted to exist. One of the main arguments in favor of this theory is that in this case the Maxwell equations would be symmetric under the exchange of electrical and magnetical quantities, and this is precisely the effect of a $\theta^D = 90°$ duality rotation [see, for example, Frankel (1979), Chaps. 9 and 10]. On the other hand, if e is the elemental electric charge and g the elemental magnetic pole, a duality rotated theory can be constructed out of the fully symmetric theory by the use of a reference duality rotation angle θ_0^D,

$$\cos\theta_0^D = e/(e^2 + g^2)^{1/2} \tag{34}$$

in which an equivalent description will have as electric charge

$$e' = (e^2 + g^2)^{1/2} \tag{35}$$

and magnetic pole

$$g' = 0 \tag{36}$$

This means that experiment can decide on the representation we are using, under duality rotation, for a theory.

In the case of the diracon fields, we can, for example, use $t_\alpha = 1$ or $t_\alpha = 0$ for the electron field, but both cases are different; if one is taken to correspond to the electron, $t_\alpha = 1$, say, the other will correspond to a $\theta^D = 90°$ duality rotated field or to monopoles of the magnetic field. These choices being, to some extent, arbitrary. The simplest first approximation is obtained when we remember that if the fields are to be identified with the known families, three families of leptons will be found (e, μ, τ), leading us to the choice $t_\alpha = 1$ for the electron field, $t_\alpha = 2$ for the muon field and $t_\alpha = 3$ for the tau field. The $t_\alpha = 4$ and the $t_\alpha = 0$ fields are equivalent and correspond, as we have already mentioned, to a magnetic monopole field.

In the following we will write $t_\alpha = t + a_\alpha$; then t corresponds to the family number as $t = n - 1$.

What we have done in practice, in equation (29), is to introduce a new type of symmetry, besides the global and local invariance which are at the origin of gauge fields which will be discussed in the next section. The new symmetry is the invariance of (29) and the gauge fields under four separate duality rotations θ_μ^B, $\mu = 0, 1, 2, 3$, which break Lorentz invariance unless all four $\theta_\mu^B = \theta^B$ (isodual case). B corresponds to set $\{a_\mu\}$. They are, for a given observer S splitted into space like $\mu = 1, 2, 3 = i$ and time like $\mu = 0$.

For observer S the a_i^B corresponding to an observable field or combination of fields are to be the same in order to maintain rotation invariance as required by experiment. Fields will be either isodual or anisodual. The isodual fields are the usual Dirac and Maxwell fields for a given nth representation. The anisodual matter fields are required to be a set of (anti-) self-dual fermion fields for (anti-) particles of the nth representation, it is (right) left handed. There will be, as discussed below, anisodual interaction fields corresponding to (anti-) self-dual vector–axial vector boson fields.

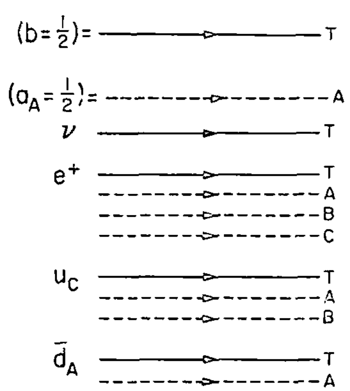

Fig. 2. Diagramatic representation of the correspondence between diracons and some elementary particles. An alternative representation is suggested of considering all fields as composite of b and a_A fields (Harari, 1979; Shupe, 1979; Keller, to be published).

In space-time it is possible to define a Galilean space and time which makes rotational invariance, in the space part, a fundamental symmetry (Reus and Keller, 1983).

Figure 2 schematically shows the first family of quarks and leptons as sets of lines conserving dual symmetry quantum numbers.

Besides the three families shown in Table I and the magnetic monopole field, we are left with one more field of the neutrino type (to be paired with the monopole field) and two more quarklike fields which have not been discussed in the literature in relation to experimental data. This fourth family is thus, this far, completely hypothetical, even the existence of the magnetic monopole is uncertain.

But a further discussion should wait for next section where the interactions are analyzed.

6. DUAL GEOMETRY THEORY OF THE INTERACTION FIELDS

The interaction fields will be assumed to obey the general relation

$$\partial_\alpha \partial^\alpha A^B = J^B \tag{37}$$

where the A are to be identified with gauge interaction fields with a set B of duality rotation constraints, arising from a source J with the same type of constraints. If the source is taken to be a current

$$J_\mu^B = \bar{\psi}^B \left(\gamma_\mu \cos \theta_\mu^B + i \gamma_5 \gamma_\mu \sin \theta_\mu^B \right) \psi^B \tag{38}$$

the field A^B will then be a vector–axial vector field. If the source is taken to be a tensor (that is a collection of four vectors), the field A^B will be a tensor field, tensor fields mix their components under duality rotations. As the sources have been required to be eigenspinors of the duality operator γ_5.

$$\gamma_5 \psi^{B'} = \pm i \psi^{B'} \tag{39}$$

the interaction fields should also be eigenvectors of the duality operator γ_5

$$\gamma_5 A^B = \pm i A^B \tag{40}$$

which will reduce (37) to a Maxwell-like equation. Here again, B corresponds to the nth set $\{a_\alpha^B\}$.

In order to write the interaction fields corresponding with the schemes accepted today, we should consider the following:

(a) Equation (37) will reduce to the Maxwell equation if a definite fundamental representation of the duality rotation angles is introduced as defined by equation (34), in which the field we have chosen to identify with the electron fields in Table I is taken to have charge and no magnetic pole as in equations (35) and (36). This reference duality rotation angle corresponds to $\theta_\alpha^D = \pi/2$, that is, full duality. Then charged particles will be those where the three spatial duality constraints are equal to $a_0 = 1$; besides the electron, only composite fields could present this characteristic, the scheme being repeated for the muon family and for the tau family.

(b) The two sets of fields called parton in each column of Table I will correspond to the quark fields, color being defined by the particular duality constrained a_i which is different from 0, then one type of quark will carry one color only and the second type of quark will carry two colors. A composite field with all three colors will be charged and a composite field without all three colors will be neutral.

(c) As will be shown in the examples of the next section, composite particles, baryon number will correspond to the number of fields where the spatial duality constraints are not symmetric, that is, where exchange of x_i and x_j, $i \neq j$, will change the representation.

(d) Weak charge, or the possibility to have a change similar to that taking a neutrino into an electron, corresponds to the possibility of simultaneously changing the three spatial duality constraints by one unit. The parton fields will allow this change among themselves in a similar way to lepton fields.

(e) We already mentioned the family number, $n = t + 1$, which of course corresponds to flavor. A flavor-changing interaction is possible which will change all duality constraints simultaneously by an integer. Combinations of interactions will reduce to the ones already listed here.

Then the interaction fields defined by equation (37) will be of three types: those which do not change the duality constraints (to be identified with the electromagnetism), second, those that do change the duality constraints (gluon, weak, and flavor-changing types of interactions), and those which, arising form the tensor part (or bivector) of the phase factors $\phi(\mathrm{x})$ in (20), will generate the necessity of a local compensating vierbein, resulting in gravitational interactions. In

$$\phi(\mathrm{x}) = \phi_S(\mathrm{x}) + \gamma_5 \phi_{PS}(\mathrm{x}) + \gamma_\mu \gamma_\nu \partial^\nu \Omega^\mu(\mathrm{x}) \tag{41}$$

the set of four Ω^μ correspond to the gravitational field.

In Table II we present the possible fields compatible with equation (37) for a vector (axial vector) current source. The equations for the gravitational

TABLE II. Possible Interaction Fields Classified According to the Duality Constraints a_μ They Carry

a_0	a_1	a_2	a_3	Possible identification
0	0	0	0	Electromagnetic
1	0	0	0	Flavor-changing field
0	1	0	0	Color-changing
0	0	1	0	fields generating the
0	0	0	1	gluon scheme
0	1	1	0	Color-changing
0	1	0	1	fields generating the
0	0	1	1	gluon scheme
0	1	1	1	Weak interaction
1	1	1	1	Flavor-changing field (Higgs?)

interaction will be discussed below, and we will not discuss in this paper the possibility of duality rotation constraints in the gravitational field Ω.

From top to bottom in Table II, we have the following:

(a) The electromagnetic field which, with the convention we have adopted above of the electron field carrying charge, exchanges energy and momentum between the interacting particles.

(b) The weak field carrying $a_i = 1$ and $a_0 = 0$, a quanta of which will be able to simultaneously change all three spatial duality constraints of an emitting or receiving particle. Changing for example an electron into a neutrino, a quark of one type into the quark of the second type or a composite particle like a proton, into a neutron as will be discussed in the next section. This field, being symmetrical on the three spatial duality constraints a_i, will carry charge itself and may be properly called W^+. For the current of the antiparticles or for the inverse interaction, a W^- field can be defined with $a_i = -1$. And from the algebra of these fields,

$$Z^0 \equiv [W^+, W^-] \tag{42}$$

a neutral field is defined. The three together present SU(2) symmetry.

(c) We find two sets of three equivalent fields which change the color of the particles as they carry one or two $a_i \neq 0$. An absorption of one quanta will change either one color or two colors simultaneously. But it must be remembered here that the fields called partons, which in fact correspond to the quarks, cannot exist alone in free space as they break rotational invariance. Then, it is not physical to consider a scheme in which one quark, say, with two colors, emits one quanta of the field being discussed, without a second quark being immediately available to receive the "color." This forces two considerations: what is immediately available? and, even if a second quark is available only some combinations of these interaction fields are possible.

The quarks we have identified with the parton fields of Table I should, in principle be massless, according to equation (29); if it were so, the coherence length of a two quark system would be, $\lambda = h/mc$, infinite, which is not possible, then composite systems, of several quarks each, must be massive, the mass being the sum of the energies of the components in the center-of-mass system, and related to some volume through this coherence length. It is within this volume that the interaction field quanta can be exchanged and the scheme of possible exchanges correspond to the eight generators of a SU(3) color symmetry, factoring out the identity, as a linear combination of

$$G_{ij} = \hat{a}_i^+ \hat{a}_j \tag{43}$$

(color j taken away and color i being given), the quanta of G_{ij} cannot be emitted outside of the composite particle as they are not rotationally invariant themselves. The strength of this interaction should, in principle, from equations (37) and (41), be the same as that of the electromagnetic field, but this will not be what will be observed in practice because only if the distance between the quarks is small, compared to the coherence length, the interaction is possible. If the two quarks were to come farther apart than this coherence length, rotational invariance would be broken and in order to restore it, a new pair of quarks will be needed, one of each near the original ones, to locally restore rotational invariance; the net effect being that a "strong" interaction will be observed, with strength equal to the energy required to create a two-quark pair and the necessary extra gluons. A symmetry constraint is transformed, from the experimental point of view, into a dynamical constraint. As this coherence length depends on the mass (energy), the strength of the color force will seem to depend on energy. Moreover, the quanta of the gluon fields, carrying color themselves, should also acquire energy in order for a coherence length $\lambda_g/n = h/mc$ to be defined as in the case of quarks. Then, the mass of a composite particle will increase with the number of quark components and with the number of gluons required to conserve color invariance. Here and in the discussion of quarks, above, mass refers to the center of mass of the composite particle, the components may remain massless in the sense of rest mass. As a first approximation the mass of the composite particle could be written

$$m = n_q m_q + n_g m_g + \Delta m \tag{44}$$

with n_q the number of quarks, n_g the number of fluons, m_q a center-of-mass system mass of the quarks, m_g a center-of-mass system mass of the gluons, and Δm the corrections arising from pairing of spins or other contributions to the binding. Our analysis is in fact in agreement with the idea of the bag model. Here we have freely used the quark terminology as accepted today; we refer the reader to the excellent bibliographic review now available in the resource letter by Greenberg (1982).

In the next section composite particles are discussed because it is clear that in the present theory it is unphysical to speak of the properties of quarks and gluons as free particles, but only of the properties they carry in an independent particle model for the elementary particles.

(d) The gravitational interaction, arising from the bivector part of the phase factor (41), because in order to compensate such gauge transformation a vierbein is needed

$$f_j = \left(f^\circ e^{-\partial\Omega}\right)_j \tag{45}$$

where the f° are locally Lorentzian tetrads. This vierbein is to be defined in such a way that the covariant derivative of the energy momentum tensor t_μ^ν defines an equation of motion: (here $u_\mu = dx_\mu/ds$, the four-velocity, $s^2 = g^{\alpha\beta}x_\alpha x_\beta$)

$$\Box_\mu t_\nu^\mu = 4\pi\sigma\frac{1}{2}\,\partial_\nu g_{\alpha\beta}u^\alpha u^\beta = 4\pi\sigma\frac{du_\nu}{ds} \tag{46}$$

requiring a condition $\partial_\nu(-g)^{1/2}g^{\mu\nu} = 0$. The metric will be given by

$$g_{\mu\nu} = \eta_{\alpha\beta}f_\mu^\alpha f_\nu^\beta = \left[\eta e^{-2\Box\Omega}\right]_{\mu\nu} \tag{47}$$

where the $\eta_{\alpha\beta}$ is a locally Lorentzian metric. This has defined a gravitational field, gauge invariant, as

$$\Phi_\mu^\nu = \partial_\mu\Omega^\nu + \partial^\nu\Omega_\mu - \delta_\mu^\nu\partial^\alpha\Omega_\alpha \tag{48}$$

which will obey, for self-consistency, the field equation

$$\Box^2\Omega = 4G\pi\left(\mathrm{T} - \tfrac{1}{2}gT\right) \tag{49}$$

with T being the tensor of the total sources. σ is the density.

Equations (45)–(49) define a selfconsistent field theory of gravitation. The metric being an exponential-type metric, as has been discussed by Yilmaz (1982) and, as discussed by Goldman (1983) in the present conference, it could be related to other metrics by a different choice of gauge.

7. COMPOSITE PARTICLES

From the preceding discussions it is now clear that there will be two types of composite particles: the ordinary ones, like atoms and nuclei which can be split into smaller components, and the heretofore called elementary particles of the hadronic type, which can be excited or transformed or may even react or decay but the components of which cannot exist as independent entities. The prototypes would be the proton for the baryons and the π^+ for the mesons.

In Figure 3 we show schematically the structure of the proton and of the neutron as composite particles, in the sense of their quark content. Also shown is one of the possible mechanisms for proton decay. Here we have used again a set of lines conserving dual symmetry quantum numbers, but we have not symmetrized schemes in relation to colors, named A, B, and C.

In Figure 3 we also show the composite structure of the π^+ meson but this time we have also included a fundamental set of gluon fields, as wavy lines which should coexist with the quark lines, in order to illustrate that the gluon content of the particles, as symmetry constraints carrying fields, should also be considered when particle excitations and decays are studied. The weak decay of the π^+, as a meson composed of quarks of the first family could be expected, from a first approximation consideration, to proceed directly into a positron and a neutrino; but if the symmetry-con-

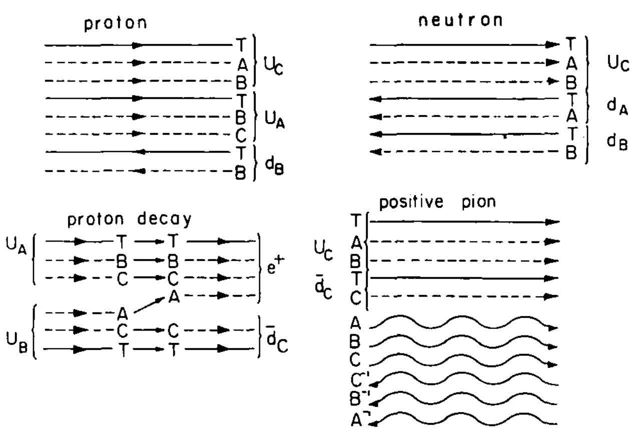

Fig. 3. Composite elementary particle fields in the diagramatic representation of Figure 2. The proton and the neutron as a three-quark system; a possible path for proton decay is also shown. For the π^+ the gluon fields are shown as wavy lines, the decay of this particle should consider the color carried by the interaction fields.

straint-carrying gluons are also considered, a weak decay into a μ^+ and a $\bar{\nu}_\mu$ is apparently more favorable. This is the case as shown by experiment. The decay of the μ^+ is, otherwise, expected to proceed via $\nu_\mu + \bar{\nu}_e + e^+$ as observed in experiment, too. In this last case there are no gluons to consider.

We have then seen two of the main roles that the gluon lines will play in the study of the physics of the composite elementary particles: as mentioned in Section 5, they will have a contribution to the mass in the center-of-mass system of the composite particle and, second, as they carry dual symmetry constraints they will favor some reaction and decay mechanisms.

Once we have accepted that quarks will only exist inside of composite particles, and that the composite particles will have to be symmetrized with respect to color, we can now think of quarks as fractional electric charge fields because they will have the correct combination of colors 1/3 or 2/3 of the time. Quarks will then be observed as fractional charge components in the electromagnetic interactions of composite elementary particles.

8. SUMMARY

We started by considering space-time and γ_5 as generator of dual symmetry in space-time. In elementary particle physics γ_5 plays a double role, projecting out helicity states and transforming vector current into axial vector current... These facts and the success of the Weinberg–Salam theory left us to search into dual symmetry as the origin of the observed schemes of quarks and leptons. We have shown that such a scheme is possible and that the immediate consequences of it agree, at least qualitatively for the time being and in the symmetry of the scheme itself, with the most relevant known facts about noncomposite and composite elementary particles.

It is interesting that the concept of color of Greenberg and Han and Nambu arises naturally from the dual-geometry symmetry scheme with all its assumed properties. Confinement in chromodynamics is of symmetry origin in the dual-geometry theory of elementary particles here presented but is transformed into a dynamical concept from rotation invariance considerations.

Composite elementary particles are then really elementary from symmetry considerations, but their structure is shown to consist of quarks and gluons as nonindependent entities which will affect the properties and reactions of the particles.

The results of this paper could be obtained from the direct postulation of equations (29) and (37) but the geometrical meaning of duality would be obscured. We believe that our approach where the double role of γ_5 is fully discussed conducts to a deeper understanding of the subject.

During the course of this conference an interesting controversy arose when Professor Dirac pointed out that a good mathematical scheme is the basis of new scientific knowledge and that later studies may bring the underlying ideas out, while Professor Wigner insisted that sound physical reasoning is a good starting point. In the case of the present paper I have found that, as usual in science, opposite points of view turn out to be complementary: the basic scheme I am using started as a physical idea years ago, then in 1979–1980 I found the mathematical scheme which may express the basic ideas and as Professor Dirac said, the discovery that dual geometry was the explanation for it came only a year later. Then ideas and mathematics have come to support each other alternately.

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